Stiffness and strength reduction of a sandwich beam with core cut-outs

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Abstract

This work is a part of the Swedish collaboration project "Lätta självbärande karossmoduler (LSK)" (Light self-supporting sandwich structures), which goal is to broaden the knowledge within the area of design and manufacturing techniques of sandwich structures. It focuses on large self-supporting sandwich structures such as house modules or bus passenger cabins.

From the view of a designer, such sandwich structures are useful for hiding supply lines as electrical wires, or hydraulic and water pipes. These can be hidden by integrating the pipes in the core material. The “defects” created by drilling holes in the core material will affect the stiffness properties and the critical load of the sandwich structure. The goal of this project work is to investigate the effect on the overall stiffness and strength of a sandwich panel which have a cut out in the core material.

In this project two idealised load cases are considered, which are pure shear and pure bending. A hole in the respective region has an effect on the stiffness properties as well as the failure mode due to its size and its vertical position. In an experimental four point bending test the critical load and the deformation of the specimens were analysed. If the defect is placed in the bending zone, the critical load is almost constant even if the size of the defect decreases. If the hole is in the shear zone, the critical load decreases.

The mode failures occurred during the tests are shear failure, local buckling as well as face wrinkling. The reason for the different mode failures are the varied properties of the used foam material.

To investigate the manner of the shear and bending stiffness, a 3D finite element (FE) model was used. As a result of the FE-calculation it can be said that the bending stiffness is not affected by the defects contrary to the shear stiffness, which decreases increasing the defect size.
Stiffness and strength reduction of a sandwich beam with core cut-outs

Contents

Abstract ........................................................................................................................................ 2

1 Introduction .................................................................................................................................. 4

2 Sandwich panel .......................................................................................................................... 5
  2.1 Material and geometrical properties .................................................................................... 5
  2.2 Specimen configuration .......................................................................................................... 7
  2.3 Manufacture of the specimens ............................................................................................... 10
  2.4 Stiffness properties ............................................................................................................... 12

3 Testing .......................................................................................................................................... 13
  3.1 Test technique ....................................................................................................................... 13
  3.2 Results of the test series ....................................................................................................... 14
  3.3 Comparison of the stress fields ............................................................................................ 25

4 Analysis of the failure modes .................................................................................................... 26
  4.1 Shear failure ......................................................................................................................... 26
  4.2 Buckling ................................................................................................................................ 28
  4.3 Face wrinkling ...................................................................................................................... 29

5 Finite element model .................................................................................................................. 31
  5.1 Model .................................................................................................................................... 31
  5.2 Stiffness properties ................................................................................................................ 33
  5.3 Results of the finite element calculation .............................................................................. 34

6 Testing and FE-calculation by comparison ................................................................................ 37

7 Conclusion .................................................................................................................................... 39

8 References .................................................................................................................................... 40

9 Appendix ..................................................................................................................................... 41

Josefine Schumacher
1 Introduction

Sandwich structure are used to increase the bending stiffness without enlarging the weight of the structure in the same magnitude. Sandwich structures are also useful regarding design and construction. For example, support piping like electrical wires or water pipes as well as heating tubes can be hidden in the core material. These defects will probably have an effect on the overall stiffness of the sandwich structure.

The main goal of the following report is to investigate how the size and the position of a drilled hole will affect the mechanical properties of the entire sandwich structure. It will be focused on the bending stiffness (D) as well as on the shear stiffness (S). A 3D finite element model is used, with which the behaviour of the stiffness properties will be explained. To demonstrate the influence on the critical load due to the defect, several specimens are manufactured and tested in a four point bending test.

Using four point bending test causes different stress fields as shear stress or bending moment in the sandwich structure. Therefore two horizontal position options are used, where the cut-out can be placed either in the shear zone or in the region of the maximal bending moment. Another aspect is the vertical position of the hole. In the vertical the defect can be located either in the centre of the core material or along the face sheet. An interesting aspect is to show how the vertical hole position will effect the failure mode. These four set-up types will be combined in such a way that four different types of specimen are created. In each case four different hole sizes will be investigated, which differ from 1 % to 21 % of the shear zone.

Also, two different materials are used for the four point bending test which are in general used in the construction industry. Both materials are cheap, but differ in the mechanical properties.
2 Sandwich panel

In this project two sandwich panels are used which were manufactured by the Swedish company “BoxModul”[1]. As illustrated in Figure 2.1, the panels consist of two steel face sheets and a foam material. One of the panels is made of the foam material called XPS, Extruded Polystyrol, while the other one includes EPS, Expanded PolyStyren. Both of them are insulating materials used also in the construction industry due to their good mechanical properties in relation to their low price. The differences between the both materials are the higher compressive strength of XPS as reported in the Appendix 9.2 as well as the slightly more brittleness of the EPS.

![Figure 2.1: Sandwich panel](image)

2.1 *Material and geometrical properties*

The mechanical properties of the different constituents are given in Table 2.1 [2] and Table 2.2, which is mentioned in the Appendix 9.1 to 9.2 and reference [3]. Regarding the foam material, which is considered to be isotropic, the tensile modulus has been calculated using the shear properties and a estimated poisson's ratio of 0.2 [4].

**Table 2.1: Data of the face material**

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_f$ [GPa]</th>
<th>$\rho_f$ [kg/m³]</th>
<th>$\nu_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>200</td>
<td>7850</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Table 2.2: Data of the core material**

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_c$ [MPa]</th>
<th>$\rho_c$ [kg/mm³]</th>
<th>$G_c$ [MPa]</th>
<th>$\tau_c$ [MPa]</th>
<th>$\nu_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPS - DC200</td>
<td>1.6</td>
<td>35</td>
<td>0.7</td>
<td>0.28</td>
<td>0.2</td>
</tr>
<tr>
<td>XPS – Ecoprim 400</td>
<td>8</td>
<td>43</td>
<td>3.7</td>
<td>0.45</td>
<td>0.2</td>
</tr>
</tbody>
</table>
The thickness relation of the sandwich structure is illustrated in Figure 2.2. The thickness of the core material is \(t_c\), while \(t_f\) represents the thickness of the face. The distance between the centre lines of the face sheets is defined as \(d\). The values for each panel are listed in Table 2.3.

![Figure 2.2: Thickness relation of the structure [4]](image)

### Table 2.3: Thickness values

<table>
<thead>
<tr>
<th>Material</th>
<th>(t_f) [mm]</th>
<th>(t_c) [mm]</th>
<th>(d) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel-EPS</td>
<td>0.6</td>
<td>39.3</td>
<td>39.9</td>
</tr>
<tr>
<td>Steel-XPS</td>
<td>0.6</td>
<td>39.8</td>
<td>40.4</td>
</tr>
</tbody>
</table>

The general size of the test specimens used in this investigation are listed in Table 2.4.

### Table 2.4: Dimensions of the specimen

<table>
<thead>
<tr>
<th>Length [mm]</th>
<th>Width [mm]</th>
<th>Thickness [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>
2.2 Specimen configuration

The specimen configurations are listed in Table 2.5. Four different specimens set-ups are used called from Type A to Type D. Each configuration is cut into four specimens, which differ in the size of defect. The size of defect varies from 1% to 21% of the shear zone, which is explained in Figure 2.5. The specimens A-L are made of the EPS foam, while the specimens M-T include XPS. For each configuration one specimen is tested, only in case of Type B the same configuration is used again using a different foam material. Two undamaged beams (UD) for each panel are tested, which act as reference beams. Their result will be compared with the results of the defect beams.

Table 2.5: Specimen configuration

<table>
<thead>
<tr>
<th>Specimen set-up</th>
<th>Specimen name</th>
<th>Radius [mm]</th>
<th>Percentage of shear zone [%]</th>
<th>Foam material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>A</td>
<td>5</td>
<td>1.2</td>
<td>EPS</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>6.6</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>10</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>15</td>
<td>10.4</td>
<td></td>
</tr>
<tr>
<td>Type B</td>
<td>E</td>
<td>5</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>6.6</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>10</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>15</td>
<td>10.4</td>
<td></td>
</tr>
<tr>
<td>Type C</td>
<td>I</td>
<td>10</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>13.3</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>20</td>
<td>9.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>30</td>
<td>20.8</td>
<td></td>
</tr>
<tr>
<td>Type D</td>
<td>M</td>
<td>10</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>13.3</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>20</td>
<td>9.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>30</td>
<td>20.6</td>
<td></td>
</tr>
<tr>
<td>Type B</td>
<td>Q</td>
<td>5</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>6.6</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>10</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>15</td>
<td>10.3</td>
<td></td>
</tr>
<tr>
<td>Type UD</td>
<td>UD (1-2)</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UD (3-4)</td>
<td>0</td>
<td>0</td>
<td>XPS</td>
</tr>
</tbody>
</table>
These specimen configurations are chosen due to the four point bending test. A schematic outline of the four point bend testing set-up is shown in Figure 2.3.

As illustrated in Figure 2.4, the shear force and as a result the shear stress between the inner and the outer supports are constant. The constant transverse force is $\pm \frac{P}{2}$. The bending moment between the inner supports is constant and equal to $\frac{PL_1}{2}$. At this section of the beam the transverse force is zero. Due to this, it exists a shear and a bending zone, where each section force is constant.
The shear zone is defined as the region between the inner and the outer support as illustrated in Figure 2.5. It can be calculated by \( A_s = d \cdot (L_2 - L_1)/2 \). The bending zone shown in Figure 2.6 is placed between the inner supports and its equation is \( A_b = d \cdot L_1 \). Depending on the dimensions of the test machine \( L_1 \) is 100 mm and \( L_2 \) is 440 mm. The values of the zones for each panel are listed in Table 2.6.

![Figure 2.5: Shear zone](image)

![Figure 2.6: Bending zone](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>Shear zone [mm(^2)]</th>
<th>Bending zone [mm(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPS</td>
<td>6783</td>
<td>3990</td>
</tr>
<tr>
<td>XPS</td>
<td>6868</td>
<td>4040</td>
</tr>
</tbody>
</table>

To see what will happen to the stiffness and the strength, two different horizontal hole positions are defined. One is placed in the bending area, while the other one is located in the shear zone as pictured in Figure 2.7.

![Figure 2.7: Horizontal position](image)
The defect can also be set either in the centre of the core or along the face sheet as given, for example, in Type A and Type C, see Figure 2.8 and 2.9 below. This position arrangement will influence the failure mode, which can be shear failure, face wrinkling or local buckling. As already mentioned the size of the cut out varies in all four types of specimens and is given as a percentage of the shear zone.

**Figure 2.8:** Type A

**Figure 2.9:** Type C

### 2.3 Manufacture of the specimens

The defects in the core material were created using a hot wire. The cutting apparatus, which consists of a wire and a transistor is shown from Figure 2.10 to 2.12 below.
Stiffness and strength reduction of a sandwich beam with core cut-outs

Based on the dimensions of the test machine, the hole positions has been determined as listed in Table 2.7. The location of the defects in the shear zone is 115 mm from the side.

<table>
<thead>
<tr>
<th>Specimen set-up</th>
<th>$x$ [mm]</th>
<th>$y$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>250</td>
<td>20</td>
</tr>
<tr>
<td>Type B</td>
<td>115</td>
<td>40</td>
</tr>
<tr>
<td>Type C</td>
<td>250</td>
<td>20</td>
</tr>
<tr>
<td>Type D</td>
<td>115</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 2.7: Origin of the defect

These different hole types are shown from Figure 2.13 to 2.16.

Figure 2.13: Type A

Figure 2.14: Type B

Figure 2.15: Type C

Figure 2.16: Type D
2.4 Stiffness properties

Stiffness properties depend on the mechanical and geometrical properties of a structure. In this case, the cross-section is rectangular and the Young's modulus of the core material is significantly lower than the Young's modulus of the face sheets - \( E_c \ll E_f \). In addition, the thickness of the face sheets is very thin compared to the core material - \( t_f \ll t_c \). The approximation for the stiffness properties \( S \) and \( D \)

\[
S = \frac{(G_c d^2)}{t_c} \quad (2.1)
\]

\[
D = \frac{(E_f t_f d^2)}{2} \quad (2.2)
\]

are found in [4]. The data of the bending stiffness and the shear stiffness for each sandwich panel is calculated and listed in Table 2.8, using the material properties given in Tables 2.1 – 2.3.

**Table 2.8: Reference stiffness data**

<table>
<thead>
<tr>
<th>Panel</th>
<th>( D ) [kNm]</th>
<th>( S ) [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPS</td>
<td>95.5</td>
<td>64.8</td>
</tr>
<tr>
<td>XPS</td>
<td>97.9</td>
<td>151.7</td>
</tr>
</tbody>
</table>
3 Testing

3.1 Test technique

The test technique used in this project is four point bending. The general mode of operating has been already given in Figure 2.3. The goal of using the four point bending test is to investigate the effect of the different holes on the critical load \( P_{\text{crit}} \) and the deflection \( w_1 \) at the load points as pictured in Figure 3.1.

![Figure 3.1: Deformation [6]](image)

The test operation is simple. As shown in Figure 3.2, the specimens are clamped in the machine. This machine is connected to a computer which records the load and the deflection as shown in Figure 3.3. Based on the data which are obtained from the recording, \( P_{\text{crit}} \) and \( w_1 \) can be determined and compared with the values of the undamaged specimens.

![Figure 3.2: Test machine](image)  ![Figure 3.3: Recording](image)
3.2 Results of the test series

The tests series have shown that different materials and different types of defects effect varied failure modes. All failure modes of the tests are listed in Table 3.1.

Table 3.1: Failure mode

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>UD (EPS)</th>
<th>UD (XPS)</th>
<th>Type A</th>
<th>Type B (EPS)</th>
<th>Type B (XPS)</th>
<th>Type C</th>
<th>Type D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear failure</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Buckling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrinkling</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Two different failure modes are exist for the reference beams. The slightly more brittle EPS UD specimen failed in shear as given in Figure 3.4, while the XPS panel failed in face wrinkling at the support, see Figure 3.5.

![Figure 3.4: UD - EPS](image)

![Figure 3.5: UD - XPS](image)

The failure mode of Type A is shown in Figure 3.6. All specimens of this type failed in shear. The failure was independent on the size of the defect.

![Figure 3.6: Type A](image)
Figure 3.7 and 3.8 show the failure modes of the Type B specimens. In case the specimens are made of EPS the failure mode is shear failure with cracks originating from the hole. The XPS specimens failed in shear, but without cracks.

![Figure 3.7: Type B - EPS](image)

![Figure 3.8: Type B - XPS](image)

The Type C specimens failed in shear as long as the defect was small, see Figure 3.9. For big cut outs like in specimen K and L, the failure mode was local face buckling as shown in Figure 3.10 and 3.11.

![Figure 3.9: Type C](image)

![Figure 3.10: Spec. K](image)

![Figure 3.11: Spec. L](image)

Type D failed in all cases in face wrinkling as illustrated in Figure 3.12.

![Figure 3.12: Type D](image)
From the recordings of the computer, load-displacement diagrams can be given. For all specimens the diagrams are separately listed in the appendix. In the following the load-displacement diagram for each configuration type are combined and compared with the corresponding undamaged beams.

**Type A**

The graphs of Type A are shown in Figure 3.13. All the lines of specimens A-D arise linear at the beginning. At the end the gradient becomes shallower and the lines offer an abrupt depression of the load, when it comes to shear failure. A similar behaviour has the undamaged beam.

![Figure 3.13: Load-displacement: Type A](image)

**Type B (EPS)**

The specimen which are made of brittle EPS and have the defect in the shear zone behave linear elastic up until failure as shown in Figure 3.14. The critical load is remarkably lower than the critical load of the reference beam.

![Figure 3.14: Load-displacement: Type B (EPS)](image)
Type B (XPS)

In Figure 3.15 the load-displacement graphs of the XPS Type B specimens are shown. At the beginning the gradient is almost linear until the core starts having plastic deformation. In the following course the curves are constant up to the point, where the face wrinkles and the core offers simultaneously shear crack.

![Figure 3.15: Load-displacement: Type B (XPS)](image)

Type C

It is seen in Figure 3.16 that the specimens of Type C have almost the same behaviour as the reference beam, only specimen K and L differ. Both of them failed in local buckling instead of shear failure as earlier mentioned.

![Figure 3.16: Load-displacement: Type C](image)
Type D

The graphs from the testing of the ductile XPS foam differ remarkably when increasing the size of the defect as shown in Figure 3.17. The bigger the defect, the lower is the critical load and the lower is the failure strain.

![Graph showing load-displacement for Type D](image)

**Figure 3.17**: Load-displacement: Type D
In Table 3.2 the results from the tests are listed. It consists of the critical load $P_{\text{crit}}$, the deflection $w_1$, as well as the relations between the critical load, respectively deformation, and the relative values of the undamaged beams.

### Table 3.2: Results

<table>
<thead>
<tr>
<th>Set-up</th>
<th>Specimen</th>
<th>$P_{\text{crit}}$ [N]</th>
<th>$w_1$ [mm]</th>
<th>$P_{\text{crit defect}}/P_{\text{crit UD}}$</th>
<th>$w_1 \text{ defect}/w_1 \text{ UD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>A</td>
<td>728</td>
<td>9.7</td>
<td>1.05</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>711</td>
<td>9.9</td>
<td>1.03</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>758</td>
<td>9.4</td>
<td>1.09</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>694</td>
<td>9.9</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>Type B</td>
<td>E</td>
<td>540</td>
<td>4.6</td>
<td>0.78</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>454</td>
<td>4.0</td>
<td>0.65</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>381</td>
<td>3.5</td>
<td>0.55</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>285</td>
<td>3.4</td>
<td>0.41</td>
<td>0.34</td>
</tr>
<tr>
<td>Type C</td>
<td>I</td>
<td>679</td>
<td>10.9</td>
<td>0.98</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>726</td>
<td>10.6</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>675</td>
<td>7.9</td>
<td>0.97</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>402</td>
<td>3.3</td>
<td>0.58</td>
<td>0.32</td>
</tr>
<tr>
<td>Type D</td>
<td>M</td>
<td>1056</td>
<td>7.6</td>
<td>0.88</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>998</td>
<td>7.7</td>
<td>0.83</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>796</td>
<td>6.7</td>
<td>0.67</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>430</td>
<td>7.3</td>
<td>0.36</td>
<td>0.90</td>
</tr>
<tr>
<td>Type B</td>
<td>Q</td>
<td>1149</td>
<td>9.7</td>
<td>0.96</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>1124</td>
<td>9.5</td>
<td>0.94</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>944</td>
<td>7.7</td>
<td>0.79</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>740</td>
<td>6.2</td>
<td>0.62</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>UD(1-2)</td>
<td>694</td>
<td>10.0</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>UD(3-4)</td>
<td>1196</td>
<td>8.1</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Josefine Schumacher
In the following the strength and the displacement for each specimen configuration dependent on the size of the defect are diagrammed.

**Type A - EPS**

In Figure 3.18 the normalized loads of Type A specimens are shown and the displacement at failure is given in Figure 3.19. The size of the defect has no influence on the strength or the displacement at failure when the hole is placed in the centre of the bending zone for this type of material configuration.

---

**Figure 3.18: Strength of specimens A-D**

**Figure 3.19: Displacement at failure of Type A**
Stiffness and strength reduction of a sandwich beam with core cut-outs

Type B – EPS

The strength of the EPS-Type B specimens decrease drastically with increasing defect size, as well as the displacement at failure as shown in Figure 3.20 and 3.21. Even a small hole will remarkably effect the deformation at failure when placed in the pure shear stress field.

![Figure 3.20: Strength of the specimens E-H](image)

![Figure 3.21: Displacement at failure of Type B (EPS)](image)
Type B – XPS

For the Type B specimens made of XPS the strength decreases almost linear with increasing defect area as shown in Figure 3.22. In contrast, the displacement, see Figure 3.23, is virtually the same as for the UD except for the largest defect.

Figure 3.22: Strength of the specimens Q-T

Figure 3.23: Displacement at failure of Type B (XPS)
Type C - EPS

As shown in Figure 3.24 the strength of the Type C specimens is almost constant up to a defect size of 10 % of the shear zone. For a larger hole the critical load is significantly reduced. The displacement illustrated in Figure 3.25 is in the beginning nearly constant and decreases at a defect area which is greater than 5 % of the shear stress field.

![Diagram of Figure 3.24: Strength of the specimens I-L]

![Diagram of Figure 3.25: Displacement at failure of Type C]
Type D - XPS

It is seen in Figure 3.26 that the strength of the Type D specimens drops almost linear with a gradient of about -3 by increasing the defect size. In contrast, the displacement of the Type D beams illustrated in Figure 3.27 is nearly constant. As seen the average reduction of the deformation is only 10%.

Figure 3.26: Strength of the specimens M-P

Figure 3.27: Displacement at failure of Type D
3.3 Comparison of the stress fields

Bending zone

In Figure 3.28 the relation between the critical load of the specimens Type A and Type C is shown compared to the load of the undamaged beams given as a function of the hole radius. Only the largest defect of Type C show a significant reduction. Neither the size nor the vertical position of the defect up to a defect area of 10% of the shear zone have an influence on the critical load. The reason of the load deviation of specimen L is the different type of mode failure. Due to the big cut-out the face sheet was unsupported against local buckling in which specimen L failed.

Shear zone

As shown in Figure 3.29 the strength decreases almost linear by increasing the defect area in case the hole is placed in the shear stress field. The vertical position of the defect has no influence on this behaviour.
4 Analysis of the failure modes

4.1 Shear failure

Shear failure occurs if \( \tau_{\text{cmax}} \leq \tau_c \), whereas \( \tau_{\text{cmax}} \) is the maximal critical shear stress and \( \tau_c \) is the shear stress causing of the four point bending test. The value for the maximal shear stress is given in Tab. 2.2, while \( \tau_c \) can be calculated by

\[
\tau_c = \frac{P}{d}
\]  

(4.1)

found in [4]. The results for the specimens are listed in Table 4.1.

**Table 4.1: Shear failure**

<table>
<thead>
<tr>
<th>Set-up</th>
<th>Specimen</th>
<th>( d ) [mm]</th>
<th>( \tau_c ) [MPa]</th>
<th>( \tau_{\text{cmax}} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>UD - EPS Type A</td>
<td>A</td>
<td></td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td></td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td></td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D</td>
<td></td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Type B</td>
<td>E</td>
<td></td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F</td>
<td></td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>G</td>
<td></td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H</td>
<td></td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>Type C</td>
<td>I</td>
<td></td>
<td>39.9</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td></td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>K</td>
<td></td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L</td>
<td></td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>UD-XPS Type D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>M</td>
<td></td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td></td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>O</td>
<td></td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P</td>
<td></td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td></td>
<td>40.4</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td></td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S</td>
<td></td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T</td>
<td></td>
<td>0.37</td>
<td></td>
</tr>
</tbody>
</table>
For the specimens from Type A and the specimen I and J the equation (4.1) matches. The maximal shear stress is greater than the shear stress obtained from the bending test. All these specimens offered shear cracks. Only the specimens K and L showed face buckling as failure mode. In these cases the face sheets were unsupported against buckling. For the beams which have the defect in the shear zone, the results of equation (4.1) are wrong. As mentioned in chapter 3.2, the Type B beams made of EPS failed in shear, while all beams made of XPS had face wrinkling as failure mode. That means, if the cut-out is placed in the shear zone, equation (4.1) can not be used in the conventional way. If the defect is placed in the shear zone the beam behaves probably more than a beam which has a lateral contraction in the shear stress field as illustrated in Figure 4.1. In this case the distance between the face centre lines $d$ has to be adapted in any way in this region.

![Figure 4.1: Lateral contraction](image)

For a brittle foam material like EPS equation (4.1) can be modified, while the diameter of the hole is subtracted from the original $d$

$$\tau = \frac{P}{(d')^2} \quad \text{for} \quad d' = d - 2r \quad (4.2)$$

The obtained result for the shear stress are listed in Table 4.2.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$d'$ [mm]</th>
<th>$\tau$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>29.9</td>
<td>0.36</td>
</tr>
<tr>
<td>F</td>
<td>26.7</td>
<td>0.34</td>
</tr>
<tr>
<td>G</td>
<td>19.9</td>
<td>0.38</td>
</tr>
<tr>
<td>H</td>
<td>9.9</td>
<td>0.57</td>
</tr>
</tbody>
</table>

For the specimens E to G the equation (4.2) is passable. If $d' \leq d/2$ as in specimen H, equation (4.2) can not be used.

For ductile foam material like XPS this modified equation is not useful either. In this case another solution should be found. However, this is not part of this project.
4.2 Buckling

As pictured in Figure 3.10 and 3.11 the failure mode of the specimens K and L was face sheet buckling. In this case the radius of the defect enfeebled the core material in such a way that the face sheet was unsupported against buckling. Compared to the critical direct stress calculated by Euler buckling $\sigma_{\text{crit}}$, the stress in the face sheet at failure caused by the critical load in the four point bending test $\sigma_{f,\text{crit}}$ was too high. For that reason the face sheet buckled.

The maximal direct stress can be computed by using formula of Euler buckling. In this project the critical load lays between the standardised cases - Case 1 and Case 2 - as illustrated below [2].

$$P_{\text{crit}} = \frac{\pi^2 E_f I}{l^2}$$

Case 1

$$P_{\text{crit}} = 4 \frac{\pi^2 E_f I}{l^2}$$

Case 2

for $l = 2R$, $I = \frac{bh^3}{12}$ and a cross section of the sheet $b = 50 \text{ mm}$ $I h = 0.6 \text{ mm}$

The critical stress is calculated by equation (4.3)

$$\sigma_{\text{crit}} = \frac{P_{\text{crit}}}{A} \text{ for } A = 30 \text{ mm}^2$$

(4.3)

The stress in the face during testing is

$$\sigma_f = \pm \frac{P \cdot (L_2 - L_1)}{2tf \cdot d}$$

(4.4)

The results of the stress in the face at failure $\sigma_{f,\text{crit}}$ and the stresses calculated by Euler buckling $\sigma_{\text{crit}}$ for both specimens K and L are listed in Table 4.3. In both cases the stress in the face sheet at failure is between the critical stress values. The assumption concerning the face buckling is justifiable.

Table 4.3: Local buckling

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$\sigma_{f,\text{crit}}$ (Four point bending) [MPa]</th>
<th>$\sigma_{\text{crit}}$ (Euler Buckling) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>95</td>
<td>37 – 148</td>
</tr>
<tr>
<td>L</td>
<td>57</td>
<td>16 – 66</td>
</tr>
</tbody>
</table>
4.3 Face wrinkling

All specimens made of XPS foam – specimens M-T - show anti-symmetrical face wrinkling. Anti-symmetrical wrinkling stress is determined by (4.5), while the maximal direct stress of the face sheet can be calculated with (4.6) [4].

\[
\sigma_{\text{crit}} = 0.5\sqrt[3]{\frac{E_f}{E_c}G_c} + 0.33 G_c \left( \frac{t_c}{t_f} \right)
\] 

(4.5)

\[
\sigma_f = \frac{\pm P \cdot (L_2 - L_1)}{(2t_fd)}
\] 

(4.6)

No face wrinkling will occur, if \( \sigma_f \leq \sigma_{\text{crit}} \). Due to the fact, that all these specimens are placed in the shear zone the value \( d \) has to be changed again as already done in chapter 'Shear failure' to calculate the stress in the face sheet.

\[
\sigma_f = \frac{\pm P \cdot (L_2 - L_1)}{(2t_f d')}
\] 

(4.7)

Using equation (4.7) above the wrinkling stress in the face sheet are calculated and listed in Table 4.4. In these test series the wrinkling stress in the face sheet has a higher magnitude than the maximal direct stress, therefore face wrinkling occurred.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( d' ) [mm]</th>
<th>( \sigma_{\text{crit}} ) [MPa]</th>
<th>( \sigma_{\text{crit}} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>UD - XPS</td>
<td>40.4</td>
<td>168</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>30.4</td>
<td>197</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>27.1</td>
<td>209</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>20.4</td>
<td>221</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>10.4</td>
<td>234</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>30.4</td>
<td>214</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>27.2</td>
<td>234</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>20.4</td>
<td>262</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>10.4</td>
<td>403</td>
<td>171</td>
</tr>
</tbody>
</table>
Even if the critical face stress value of the undamaged beam is slightly lower than the critical wrinkling stress value, the failure mode in this case was wrinkling as well as shown in Figure 4.2.

Figure 4.2: Face wrinkling
5 Finite element model

Using a finite element (FE) model, a rating shall be given how the bending stiffness (D) and the shear stiffness (S) of the sandwich beam will change with increasing the size of the hole and by altering its position. Based on geometrical properties of the specimens already given in chapter 2, three main finite element models were created. A four point bending test simulation is accomplished and the obtained results are discussed and compared.

5.1 Model

For this project three 3D finite element models are created by using the commercial FE-code ABAQUS [7]. Creating three main models was necessary due to the meshing and the boundary condition in ABAQUS. All models have the same assembly and consist of a core part and two face sheets. To investigate the behaviour of the bending stiffness and the shear stiffness a four point bending test was simulated. Four point bending test is a symmetrical testing mode, where the maximal deflection will occur in the middle of the beam. If the defect is placed in the bending zone, the maximal deflection will still be in the middle of the beam. In case the the defect is only set in one of the critical shear zones, the maximal deflection will not be in the middle of the beam any more. Therefore in the models having the defects in the shear zone the defects were simulated in both shear zones. To simplify the FE-calculation only half of the beam are created and completed by using a symmetry line as pictured in Figure 5.1.

![Figure 5.1: Symmetric share of UD](image)

Using model 1, specimen Type A, Type B and the undamaged beam (UD) can be simulated as shown from Figure 5.1 to 5.3. Only the hole position and the boundaries condition have to be changed. At all models, the size and the position of the defect can easily be changed by entering the desired values at the respective positions as shown in the Figure 5.4. Consequently, all different types of holes given in chapter 2.1 can be modeled.

![Figure 5.2: Model 1 - Type A](image)

![Figure 5.3: Model 1 - Type B](image)
Furthermore, model 2 simulates the specimen Type C, while model 3 stands for the Type D as given in Figure 5.5 and Figure 5.6.

To simulate a four point bending test, four supports are also needed. Due to the symmetric case only two supports have to been created. The positions of the support can also easily be changed. Therefore data points are created and placed by using coordinates where the supports are supposed to be later. The positions of the supports needed for the four point bending test are listed in Table 5.1.

<table>
<thead>
<tr>
<th>Support</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support 1</td>
<td>30</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>Support 2</td>
<td>200</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>
5.2 Stiffness properties

In the four point bending test at the finite element calculation, a deformation of -10 mm was set. This fixed deformation causes reaction forces on the supports. By means of them a conclusion of the bending stiffness can be drawn. Using simple formula of sandwich theory, see equation (5.1) and (5.2), the shear stiffness can be determined [4]. Equation (5.1) represents the deformation at the load points \( w_1 \), while equation (5.2) demonstrates the deflection in the middle of the beam \( w_2 \).

\[
\begin{align*}
    w_1 &= w \left( \frac{(L_2-L_1)}{2} \right) = \frac{P \cdot (L_2-L_1)^2 \cdot (L_2-2L_1)}{24D} + \frac{PL_2}{2S} \quad (5.1) \\
    w_2 &= w \left( \frac{L_2}{2} \right) = \frac{P \cdot (L_2-L_1) \cdot (2L_2^2+2L_1L_2-L_1^2)}{24D} + \frac{PL_2}{2S} \quad (5.2)
\end{align*}
\]

To obtain the shear stiffness \( S \), both equations are used to create a formula for the shear stiffness. This equation given in (5.3) is a function of the standardised reaction load \( P \), the geometrical absolute terms \( C_1 \) and \( C_2 \) and the deflections \( w_1 \) and \( w_2 \).

\[
S = \frac{(PL_2(C_1-C_2))}{2(C_1w_2-C_2w_1)} \quad (5.3)
\]

The values \( C_1 \) and \( C_2 \) used in this formula are shown in equation (5.4) and (5.5).

\[
\begin{align*}
    C_1 &= (L_2-L_1)^2(L_2+2L_1) \quad (5.4) \\
    C_2 &= (L_2-L_1)(2L_2^2+2L_1L_2-L_1^2) \quad (5.5)
\end{align*}
\]

The values of \( w_1 \) and \( w_2 \) are chosen from the lower face sheet of the model. Due to the material compression inside the structure the values of deformation at the lower face are 3% less than the deflection on the top face. The trend of the shear stiffness is still the same. The reason for choosing the deflection values from the lower face is a local deformation at the load point at the top face in the ABAQUS calculation. The influence of this phenomena can be neglected concerning the shear stiffness. The shear stiffness behaviour is still granted. In the case of analysing the bending stiffness this influence of the local deformation can not be disregard. Therefore the reaction forces need to be considered. These are a result of the combination of the stiffness properties. If the the shear stiffness is known, a tendency of the bending stiffness can given by means of reaction forces.
5.3 Results of the finite element calculation

As shown in Figure 5.7, the shear stiffness for the specimen of Type A and Type C is constant independent of the defect size. Neither the size of the hole nor its vertical position influences the behaviour of the shear stiffness.

In contrast, in case the defect is placed in the shear zone, the shear stiffness decreases with growing the cut-out as shown in Figure 5.8.

As seen in the diagrams above the vertical hole position has no influence on the behaviour of the shear stiffness.
Regarding the absolute values from the shear stiffness when the defect is placed in the shear zone a trend line can be given as shown in Figure 5.9.

The equation for this line is

$$S = 269.4 e^{-0.0437x} \text{ for } x = \frac{(2r^2\pi)}{(d(L_2 - L_1))} \quad (5.1)$$
Reaction forces result from the stiffness properties and they are proportional to them. Reaction forces are independent of the local deformation of the beam. Therefore, the reaction forces are taken to give a trend of the behaviour of the bending stiffness. Using a fixed deformation at the supports of -10 mm in the ABAQUS calculation, the reaction forces reflect the complete stiffness of the entire structure. As shown in Figure 5.10 the reaction forces for the defect in the bending zone is constant independent of the vertical position of the hole. Due to the shear stress, which is zero in this region, the stiffness of the beam is not changed when the cut out is in the bending zone. Consequently, the size and the vertical position of the defect have no influence on the reaction forces, respectively on the bending stiffness, if the hole is placed in the bending zone.

![Figure 5.10: Reaction force in bending zone](image)

If the defect is placed in the shear zone, the critical load drops when the size of the defect increases as shown in Figure 5.11. Only the fact that the hole is set in the shear zone lets the critical load according to this the shear stiffness fall.

![Figure 5.11: Reaction force in shear zone](image)
6 Testing and FE-calculation by comparison

To compare the test results with the ABAQUS calculation, the load graphs are considered. Except for the diagram of the Type C, the trends of the load curves for each specimen type look almost identical as pictured in Figure 6.1 to 6.4. In case the defect is set in the bending zone the load trends are extensively continuous. In case of Type A both the critical load from the testing and the reaction forces from the ABAQUS calculation are constant independent of the size of the defect. They are also close to the value of the undamaged beam as shown in Figure 6.1.

![Figure 6.1: Strength relation – Type A](image)

The load relations of Type C from the tests and from the ABAQUS are constant, except the critical load of the specimen L differs as illustrated in Figure 6.2. In this case the ABAQUS value is not in accordance with the testing value due to the linearity of the FE program. ABAQUS neglects nonlinear phenomena like face sheet buckling, which occurred in specimen L.

![Figure 6.2: Load of the specimens I-L](image)
If the defect is placed in the shear zone both the critical load and the reaction forces decrease by increasing the size of the defect in the same manner, see Figure 6.3 and 6.4. Only the grade of aberration is in case of the ABAQUS calculation marginal lower.

Figure 6.3: Strength relation – Type B

Figure 6.4: Strength relation – Type D
7 Conclusion

The aim of this project work was to investigate the effect on the strength and the overall stiffness of a sandwich beam due to a cut-out in the core material. Based on the behaviour of the section forces caused by the four point bending test, two horizontal positions of the defect were defined. A hole could be set either in the shear zone between the inner and the outer support or in the bending zone in the middle of the beam. In addition two different vertical place options were set as well, to analyse the influence on the failure mode. Thus a defect was set either in the centre of the core or along the face sheet. For each configuration type, four specimens were used, where the size of the defect was changeable and the defect area accounted from 1% until 21% of the shear area. Furthermore a second foam material was used for one of the configurations to see what will be the effect concerning the critical load.

To characterise the general attitude of the critical load in respect of the defect size and position, all specimens were tested in four point bending. To explain the shear stiffness (S) and the bending stiffness (D), a 3D finite element model was created and evaluated.

As a resumption can be said that in case of a brittle foam material like EPS the size of the defect, when placed in the bending zone, has no influence on the critical load as long as the area of defect does not capture more than 10% of the shear area. In this case face buckling will occur instead of the shear failure and the critical load will be lower. If the defect is located in the shear zone, the core withstands lower shear stress independent of the material. Hence, the critical load decreases depending on the defect size, which also means that the shear stiffness is affected by the hole size.

From the ABAQUS calculation follows that neither the position nor the the size of the defect influences the bending stiffness. Only the shear stiffness and according to this the critical load will be effected due to a cut-out in the core material.

In this project only one specimen for each hole configuration, except for the defect in the centre of the shear zone, are tested and evaluated. To verify the result of this project, more tests with the same parameters should be performed. As shown in the test result, different foam materials with different properties will influence the findings. Therefore test series with varied core materials should be arranged.
8 References

[7] ABAQUS Documentation version 6.4
9 Appendix

Appendix 9.1: Data sheet of EPS [8]
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.2: Data sheet of XPS [9]
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.3: Load-displacement diagram - specimen A

Appendix 9.4: Load-displacement diagram - specimen B
Appendix 9.5: Load-displacement diagram - specimen C

Appendix 9.6: Load-displacement diagram - specimen D
Appendix 9.7: Load-displacement diagram - specimen E

Appendix 9.8: Load-displacement diagram - specimen F
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.9: Load-displacement diagram - specimen G

Appendix 9.10: Load-displacement diagram - specimen H
Appendix 9.11: Load-displacement diagram - specimen I

Appendix 9.12: Load-displacement diagram - specimen J
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.13: Load-displacement diagram - specimen K

Appendix 9.14: Load-displacement diagram - specimen L
Appendix 9.15: Load-displacement diagram - specimen M

Appendix 9.16: Load-displacement diagram - specimen N
Appendix 9.17: Load-displacement diagram - specimen O

Appendix 9.18: Load-displacement diagram - specimen P
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.19: Load-displacement diagram - specimen Q

Appendix 9.20: Load-displacement diagram - specimen R
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.21: Load-displacement diagram - specimen S

Specimen S

Appendix 9.22: Load-displacement diagram - specimen T

Specimen T
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.23: Load-displacement diagram - specimen UD1

Specimen UD1

Appendix 9.24: Load-displacement diagram - specimen UD2

Specimen UD2
Stiffness and strength reduction of a sandwich beam with core cut-outs

Appendix 9.25: Load-displacement diagram - specimen UD3

Appendix 9.26: Load-displacement diagram - specimen UD4